Pixpolar's Modified Internal Gate (MIG) image sensor technology offers numerous benefits over traditional Charge Coupled Device (CCD) and Complementary Metal Oxide Semiconductor (CMOS) image sensors like e.g. that it enables an arrangement wherein the desired ISO value can be chosen afterwards. In this white paper, however, only low light image quality is analyzed and compared between traditional and MIG image sensors.

EXECUTIVE SUMMARY

A low light image quality comparison between Pixpolar's MIG image sensors and traditional image sensors can be made based on information given in chapter "Performance comparison between traditional and MIG image sensors". The term "traditional image sensors" refers to CCD and CMOS image sensors. In below tables 1 & 2 numerical values are presented for such a comparison under specific low light circumstances described in afore said chapter.

TABLE 1 & 2. EXPOSURE TIME COMPARISONS BETWEEN TRADITIONAL AND NDCDS READOUT

LOSSLESS ROLL CORRECTION	$\dfrac{t_{trad}}{t_{MIG}}$	LOSSY ROLL CORRECTION	$\dfrac{t_{trad}}{t_{MIG}}$
Non-optimized SNR, subject	1.46	**Non-optimized SNR, subject**	1.49
Non-optimized SNR, background	2.21	**Non-optimized SNR, background**	2.11
Optimized SNR, subject	1.54	**Optimized SNR, subject**	1.61
Optimized SNR, background	2.23	**Optimized SNR, background**	2.31

Tables 1 & 2 comprise ratios of exposure times required to reach a certain image quality in low light when the use of flash is not preferred or when at least one subject is out of flash reach. The tables 1 and 2 correspond to different ways of handling of image blur which is present during the long exposure times that are required in low light. These blur handling schemes are referred to as lossless roll correction and lossy roll correction which will be explained later on in the text.

In tables 1 & 2 the abbreviation "trad" is used for traditional image sensors. In the comparison the only difference between the traditional and MIG image sensors is that in the traditional sensors a destructive readout procedure (DCDS) is utilized whereas in MIG sensors the readout procedure is non-destructive (NDCDS). This difference is, however, profound since in low light the exposure times corresponding to DCDS and NDCDS vary significantly. For example, the ratio 1.61 for a subject means that when a 10 second exposure time is used in a camera equipped with a MIG sensor one has to use an exposure time of 16.1 seconds in a camera equipped with a traditional sensor in order to reach the same image quality of the subject.

In the comparison different fixed signal generation levels are used for subject and background areas. This does not actually correspond to reality since even in adjacent pixels corresponding to different RGB (Red Green Blue) colors the signal generation levels differ considerably from each other (only in white pixels the values could correspond to a larger image area). The point is, however, that in order to compose the colors (and details) correctly in low light the ability to detect very small signal levels is required – the fixed signal generation rate values are used to give an overall estimate for this ability.

Another aspect is that the frame rate of the traditional sensor should be optimized according to average frequencies at which enough subject movement and/or camera roll movement is introduced to spoil a frame. In the comparison estimated fixed values are used for these frequencies. The latter movement depends, however, on the firmness of the grip and may differ a lot between different photographers. The former movement depends, on the other hand, considerably on the position of the subject (e.g. sitting vs. standing) and may also differ considerably between different subjects (e.g. child vs. adult).

The frame rate optimization of traditional sensors is naturally not trivial and especially so in case there are many subjects in the scenery in different positions. Thus the performance of traditional sensors can actually be much worse than what is described in the tables 1 & 2. A great benefit of the MIG sensor is that no frame rate optimization is required since the sensor can always be operated with maximum reasonable frame rate without impeding the image quality unlike the traditional sensors.

BACKGROUND INFORMATION

This white paper enables exact numerical comparison to be made on low light image quality between Pixpolar's novel Modified Internal Gate (MIG) image sensor technology and traditional image sensor technologies comprising Charge Coupled Device (CCD) and Complementary Metal Oxide Semiconductor (CMOS) image sensor technologies.

In all image sensors accurate readout of the signal necessitates the use of a Correlated Double Sampling (CDS) readout procedure. The problem in traditional image sensors is that the signal is destroyed in the CDS readout which is hereby referred to as Destructive CDS (DCDS). In MIG image sensors, on the other hand, due to Non-Destructive CDS (NDCDS) readout ability the signal is not destroyed and thus can be readout accurately as many times as desired. It should be noted, however, that in MIG sensors it is possible to freely choose between DCDS or NDCDS readout procedures.

The problem in low light photography is that there is only little light available. In order to obtain decent quality images, i.e., in order to achieve high enough Signal to Noise Ratio (SNR) there are two possibilities, either to use a flash or a long exposure time. Both of these methods can naturally also be combined.

In photography flash has traditionally been used for improving low light image quality. The problem with flash is, however, that it can only illuminate subjects/objects that are at close proximity to the camera. Besides, the flash consumes a lot of power. Yet another problem is that images obtained with flash have typically an unnatural appearance and especially so if a direct flash is used that is attached to the camera which is the case e.g. in mobile phones. By utilizing a powerful indirect flash (or beneficially multiple synchronous indirect flashes) situated apart from the camera and suitable reflectors one can improve the image appearance tremendously but this is hardly a possibility for mobile phones.

The benefit of long exposure time in low light is that one can harvest plenty of light from subjects and objects that are situated out of flash reach as well as from the background. Another benefit is also that the image appearance is natural. The problem in long exposure time images is, on the other hand, that the image quality is easily spoiled by camera and/or subject movement induced image blur.

In order to deal with camera and subject induced image blur the image sensor should be readout with a fast enough frame rate. By removing the frames or frame areas which are spoiled by image blur and by performing suitable translations and rotations to the remaining frame areas and frames it is possible to overlay and merge the frames together in a manner that a blur free long exposure time image is obtained. The problem is, however, that in order to cope with camera movement induced image blur the sensor should be readout with a relatively high frame rate unless the camera is equipped with an Optical Image Stabilizer (OIS, e.g. Nokia 925, HTC One, LG G2).

The OIS comprises at least two axial angular velocity sensors for monitoring camera's angular pitch and yaw rotations as well as of floating lens or sensor shift arrangement to counteract the pitch and yaw rotations. The roll rotation of the camera cannot, however, be counteracted with an OIS. There are two different ways to deal with the roll rotations. In the first one the sensor is readout with a constant frame rate and frames that are spoiled by roll motion are simply thrown away. This is hereby referred to as lossy roll correction since part of information is destroyed due to the roll rotation.

Another way to deal with the camera roll motion is to readout the sensor before the roll motion results in image blur which is hereby referred to as lossless roll correction since in this method no signal is lost due to

roll rotation. In order to realize the lossless roll correction a three axial (pitch, yaw, & roll) angular velocity sensor as well as fast enough image capture has to be deployed. There are already mobile phones equipped with three axial angular velocity sensors (e.g. Nokia 925). On the other hand, there are several ways how fast enough image capture can be realized.

One way to realize fast enough image capture is to use a mechanical shutter which can be turned on immediately when the roll movement exceeds a preset limit. The mechanical shutter is, however, problematic. First of all, during the time the shutter is closed photons are lost which is a problem in low light. Secondly, the mechanical shutter cannot be used at high frame rates like e.g. in video mode.

Besides the mechanical shutter another way to enable fast enough image capture is to provide the image sensor with global shutter functionality. The problem is, however, that the global shutter functionality should not increase the noise since otherwise the image quality in low light would be spoiled. Consequently, global shutter and CDS readout operation should be simultaneously enabled which is the case in progressive interline transfer CCD image sensors.

The downsides of the CCD image sensors in mobile phone applications are high power consumption, high price, and poor integrability (more chips are required than in CMOS image sensors) and thus CMOS image sensors are more or less solely used in mobile phone cameras. Unfortunately, however, the present CMOS image sensors of mobile phone cameras do not enable simultaneous global shutter and CDS readout operation which is naturally a problem in low light. It would actually be possible to design a CMOS image sensor enabling simultaneous global shutter and CDS operation but this would double the pixel size and is therefore not used.

Beside the global shutter functionality another way to enable fast enough image capture in CMOS image sensors is to provide them with fast enough frame readout. The best way to realize this is to use Back-Side Illuminated (BSI) CMOS image sensors which are face to face bonded to a readout chip. Such stacked BSI CMOS image sensors (e.g. Sony Exmor RS) are already in the market in some high end phones (e.g. Sony Experia Z) in order to provide high frame readout speed for High Dynamic Range (HDR) images and video. Consequently, lossless roll correction should already be feasible for high end mobile phones.

It should be noted that the benefit of lossy roll correction over lossless roll correction is that it requires neither global shutter functionality nor face to face bonded image sensor and readout chips. The downside of it is, however, that more signal is wasted and thus the exposure time will be slightly longer.

In the next pages low light image quality comparisons are made between MIG and traditional sensors. The calculations are based on equations presented later on in the text and correspond to different circumstances (tripod, camera is held in hand, motionless scenery, subjects in the scenery, lossless roll correction, lossy roll correction, multiple DCDS readout, multiple NDCDS readout).

PERFORMANCE COMPARISON BETWEEN TRADITIONAL AND MIG IMAGE SENSORS

Low light image quality comparison between MIG sensors and present CCD and CMOS image sensors can be made with the help of equations (1) – (82) presented later on in the text. Two examples are given wherein subjects are photographed, the camera is held in hand, and no flash is utilized. In both of the examples it is assumed that the properties of the sensors under comparison are similar except that in CCD/CMOS sensors multiple DCDS readout is utilized whereas in the MIG sensor multiple NDCDS readout is utilized. The first example corresponds to lossless roll correction and the second to lossy roll correction.

In both MIG and CCD/CMOS sensors the following assumptions are common:
-read noise $R = 2$
-the dark signal rate per pixel per second $D = 10$
-signal generation rate per pixel per second corresponding to subject area $\mu I_s = 40$
-signal generation rate per pixel per second corresponding to background area $\mu I_b = 10$
-average frequency at which enough roll is introduced to spoil a frame $f_r = 3$
-average frequency at which enough subject movement is introduced to spoil a frame $f_s = 5$

In CCD/CMOS sensors it is assumed that the frame rate f of the sensor is optimized for subjects according to afore described parameters. In MIG sensors, on the other hand, it is beneficial to use as high frame rate as possible. In the next examples it is assumed that in the MIG sensors the frame rate $f = 60$ which can be justified by the fact that it corresponds to the frame rate required by 30 Hz HDR video.

The equations for Signal Noise Ratio SNR can be expressed in the following form

$$SNR = G\sqrt{t} \, , \tag{C1}$$

wherein t is the exposure time and G represents the time independent part of SNR. With the help of (C1) a comparison between different integration times required to reach certain SNR can be made with the following equation

$$\frac{t_{DCDS}}{t_{NDCDS}} = \frac{G^2_{NDCDS}}{G^2_{DCDS}} \, . \tag{C2}$$

CALCULATIONS; HAND HELD CAMERA, SUBJECTS IN THE SCENERY, & LOSSLESS ROLL CORRECTION

A comparison between the required exposure times in CCD/CMOS and MIG sensors to reach a certain SNR can be made according to equations (45) – (47) and (77) – (79). Two cases are analyzed; in the first one no pixel specific SNR optimization is performed whereas in the second one pixel specific SNR optimization is utilized. The actual exposure time comparison according to (C2) is presented in table 1 of the executive summary chapter.

No pixel specific SNR post-optimization, DCDS readout, subject area

Due to the lack of optimization the pixel specific optimization parameter $T_{s00} = 0$. With this condition the SNR in the subject area is maximized when $f = 11$ which is calculated at an accuracy of 0.1. These values correspond to $G_{s11} = 3.19$.

No pixel specific SNR post-optimization, DCDS readout, background area

In case a very small number for f_s is used, the same equations can be utilized for background as for subject area. Consequently, $f_s = 0.001$ is utilized for background. Due to the lack of optimization the pixel specific optimization parameter $T_{b00} = 0$. The frame rate in the background is the same as in the subject area, i.e., $f = 11$. These values correspond to $G_{b11} = 1.19$.

No pixel specific SNR post-optimization, NDCDS readout, subject area

Due to the lack of optimization the pixel specific optimization parameters $T_{s1} = 0$, $T_{s2} = 0$, $m_{s1} = 1$, and $m_{s2} = 1$. These values correspond to $G_{s12} = 3.85$.

No pixel specific SNR post-optimization, NDCDS readout, background area

In case a very small number for f_s is used, the same equation can be utilized for background as for subject area. Consequently, $f_s = 0.001$ is utilized for background. Due to the lack of optimization the pixel specific optimization parameters $T_{b1} = 0$, $T_{b2} = 0$, $m_{b1} = 1$, and $m_{b2} = 1$ wherein the only relevant parameter for background is T_{b1}. These values correspond to $G_{b12} = 1.77$.

Pixel specific SNR post-optimization, DCDS readout, subject area

The SNR in the subject area is maximized when $T_{s00} = 0.0278$ and $f = 10.8$. These values correspond to $G_{s11} = 3.23$.

Pixel specific SNR post-optimization, DCDS readout, background area

In case a very small number for f_s is used, the same equations can be utilized for background as for subject area. Consequently, $f_s = 0.001$ is utilized for background. The frame rate in the background is the same as in the subject area, i.e., $f = 10.8$. The SNR in the background area is optimized when $T_{b00} = 0.0361$. These values correspond to $G_{b11} = 1.22$.

Pixel specific SNR post-optimization, NDCDS readout, subject area

The SNR in the subject area is maximized when $T_{s1} = 3$, $T_{s2} = 5$, $m_{s1} = 0.0350$, and $m_{s2} = 0.0833$. These values correspond to $G_{s12} = 4.01$.

Pixel specific SNR post-optimization, NDCDS readout, background area

In case a very small number for f_s is used, the same equation can be utilized for background as for subject area. Consequently, $f_s = 0.001$ is utilized for background. The SNR in the background area is maximized when $T_{b1} = 0.105$, ($T_{b2} = 0.208$, $m_{b1} = 7$, and $m_{b2} = 13$). These values correspond to $G_{b12} = 1.82$.

CALCULATIONS; HAND HELD CAMERA, SUBJECTS IN THE SCENERY, & LOSSY ROLL CORRECTION

A comparison between the required exposure times in CCD/CMOS and MIG sensors to reach a certain SNR can be made according to equations (31) & (32) and (34) – (36). Two cases are analyzed; in the first one no pixel specific SNR optimization is performed whereas in the second one pixel specific SNR optimization is utilized. The actual exposure time comparison according to (C2) is presented in table 2 of the executive summary chapter.

DCDS readout, subject area

The SNR in the subject area is maximized when $f = 14.8$. This value corresponds to $G_{s9} = 2.92$.

DCDS readout, background area

The value $f_s = 0$ is used for the background area. The frame rate in the background is the same as in the subject area, i.e., $f = 14.8$. These values correspond to $G_{b9} = 1.02$.

No pixel specific SNR post-optimization, NDCDS readout, subject area

Due to the lack of optimization the pixel specific optimization parameter $m_s = 1$. This values corresponds to $G_{s10} = 3.57$.

No pixel specific SNR post-optimization, NDCDS readout, background area

The value $f_s = 0$ is used for the background area. Due to the lack of optimization the pixel specific optimization parameter $m_b = 1$. These values correspond to $G_{b10} = 1.48$.

Pixel specific SNR post-optimization, NDCDS readout, subject area

The SNR in the subject area is maximized when $m_s = 4$. This value corresponds to $G_{s10} = 3.71$.

Pixel specific SNR post-optimization, NDCDS readout, background area

The value $f_s = 0$ is used for the background area. The SNR in the background area is maximized when $m_b = 10$. These values correspond to $G_{b10} = 1.55$.

EQUATIONS FOR LOW LIGHT IMAGE QUALITY UNDER DIFFERENT CIRCUMSTANCES

In the derivation and utilization of the equations in next subsections please refer also to Appendix when appropriate.

TRIPOD, MOTIONLESS SCENERY, SINGLE READOUT

When the camera is attached to a tripod and there is no motion in the scenery there will be naturally no image blur. In case only a single readout is taken the image quality (i.e. the SNR) can be represented by the following equation

$$SNR_1 = \frac{\mu I t}{\sqrt{(\mu I + D)t + R^2}} \approx \frac{\mu I}{\sqrt{\mu I + D}} \sqrt{t} \,, \tag{1}$$

wherein μ is the Quantum Efficiency (QE), I is the amount of photons striking the pixel area per second, i.e. the photon flux per pixel, D is the dark signal rate per pixel per second, R is the read noise, and t is the exposure time. The approximation applies when the term $(\mu I + D)t$ is much larger than R^2.

TRIPOD, MOTIONLESS SCENERY, & MULTIPLE DCDS READOUT

When the camera is attached to a tripod and there is no motion in the scenery the image quality of DCDS readout can be presented by the equation

$$SNR_{2DCDS} = \frac{\mu I t}{\sqrt{(\mu I + D + R^2 f)t}} = \frac{\mu I}{\sqrt{\mu I + D + R^2 f}} \sqrt{t} \,, \tag{2}$$

wherein f is the frame rate (i.e. readout frequency) of the image sensor. The disadvantage of the multiple readout procedure is higher noise when compared to single readout. The advantage is, however, that the exposure time can be set afterwards.

TRIPOD, MOTIONLESS SCENERY, & MULTIPLE NDCDS READOUT

When the camera is attached to a tripod and there is no motion in the scenery the image quality of NDCDS readout can be presented by the equation

$$SNR_{3NDCDS} = \frac{\mu I t}{\sqrt{(\mu I + D)t + R^2}} \approx \frac{\mu I}{\sqrt{\mu I + D}} \sqrt{t} \,. \tag{3}$$

The advantage of the multiple readout procedure when compared to single readout is that the exposure time can be set afterwards without increasing the noise.

HAND HELD CAMERA, MOTIONLESS SCENERY, LOSSY ROLL CORRECTION, & MULTIPLE DCDS READOUT

In lossy roll correction some of the multiple frames composing the long exposure time image are spoiled by image blur which is caused by hand movements. In case the camera is held in hand and the scenery is motionless the image quality of lossy roll correction in DCDS readout is represented by equation

$$SNR_{4DCDS} = \frac{\mu I t e^{-f_r/f}}{\sqrt{(\mu I + D + R^2 f) t e^{-f_r/f}}} = \frac{\mu I}{\sqrt{\mu I + D + R^2 f}} e^{-f_r/2f} \sqrt{t} \,, \tag{4}$$

wherein f_r corresponds to the average frequency at which a frame is spoiled by roll motion. The optimal frame rate in equation (4) maximizes the SNR and corresponds to the zero value of the derivative of the equation (4). The derivative of (4) is

$$\frac{\partial}{\partial f} SNR_{4DCDS} = -\frac{R^2 \mu I \sqrt{t} e^{-f_r/2f}}{2f^2 (\mu I + D + R^2 f)^{3/2}} \left[\left(f - \frac{f_r}{2} \right)^2 - \frac{f_r^2}{4} - \frac{f_r(\mu I + D)}{R^2} \right] . \tag{5}$$

which is zero at

$$f_{4optimum} = \frac{f_r}{2} \left(1 + \sqrt{1 + 4\frac{\mu I + D}{R^2 f_r}} \right) \tag{6}$$

and therefore represents the optimum frame rate at certain signal generation rate μI corresponding to either green, red, blue, and possibly white pixel. The problem with the frame rate optimization is naturally that the frame rate has to be preset according to f_r and μI. The former may vary considerably in different occasions and between different people which may hold the camera. The latter may, on the other hand, vary considerably throughout the image area as well as between pixels of different colors. The optimum value for the frame rate would be a weighted average taking into account the intensities in all of the pixels and the assumed roll correction rate which is more or less an impossible task to be performed fast enough for practical photography.

HAND HELD CAMERA, MOTIONLESS SCENERY, LOSSY ROLL CORRECTION, & MULTIPLE NDCDS READOUT

In this case it is assumed that during a k frames long time period ($0 \leq t \leq k/f$) the image blur is below a threshold and that after $k + 1$ frames enough blur is introduced overcome the threshold. Such a period ($0 \leq t \leq k/f$) is referred to as blur free period. It is hereby assumed that the information according to the $k + 1$ frame [corresponding to a time period of $k/f \leq t \leq (k + 1)/f$] is thrown away and that a next investigation period is started from the $k + 2$ frame [i.e., from the time point $(k + 1)/f \leq t$ onwards]. In this manner there is at least one "deleted" frame in between two blur free periods. This means that the two blur free periods are completely uncorrelated which simplifies the model to be used.

In order to minimize the signal loss one could start the next investigation period already from the $k + 1$ frame (i.e., from the time point $k/f \leq t$ onwards). This would mean, however, when a blur free period starts immediately after another one the two blur free periods would be correlated.

In order to keep things simple it is also assumed that the first frame of the blur free period is subtracted from the last one and that the information in the intermediate frames is thrown away. One should note, however, that the read noise could be reduced by performing regression analyses on all of the frames belonging to the same blur free period. The downside of the regression analyses is, on the other hand, that the corresponding equations would be more complicated and thus it is omitted.

Due to above reasons the SNR equations corresponding to multiple NDCDS readout underestimate somewhat the actual SNR. According to the previously explained procedure the equation for SNR corresponding to NDCDS readout and lossy roll correction can be obtained in the following way. The

probability for having k frames long blur free period which is followed by n subsequent non-successful frames is given by the equation

$$P_1(k,n) = \int_{k/f}^{(k+1)/f} f_r exp(-f_r t)dt \cdot \prod_{p=0}^{n-1}\left\{\int_{(k+p)/f}^{(k+p+1)/f} f_r exp\{-f_r[t-(k+p)/f]\}dt\right\}/[1-exp(-f_r/f)]$$

$$= [exp(-f_r/f)]^k[1-exp(-f_r/f)]^n ,$$ (7)

wherein the last division in the first row is included so that the interval $k/f \le t \le (k+1)/f$ would not be taken twice into account. Thus the overall probability of having a k frames long blur free period is

$$P_2(k) = [exp(-f_r/f)]^k \sum_{n=1}^{\infty}[1-exp(-f_r/f)]^n = [exp(-f_r/f)]^k \frac{1-exp(-f_r/f)}{exp(-f_r/f)} .$$ (8)

The square of noise corresponding to (8) can be represented by the equation

$$\tilde{N}_1(k) = P_2(k)\left[(\mu I + D)\frac{k}{f} + 2R^2\right] = [exp(-f_r/f)]^k \frac{1-exp(-f_r/f)}{exp(-f_r/f)}\left[(\mu I + D)\frac{k}{f} + 2R^2\right] .$$ (9)

By assuming that we choose only signal that originates from at least m frames long blur free periods the square of the overall noise can be expressed with the help of (A2) as

$$\tilde{N}_2(m) = \sum_{k=m}^{\infty} \tilde{N}_1(k) = \frac{1-exp(-f_r/f)}{exp(-f_r/f)}\left\{\frac{\mu I + D}{f}\sum_{k=m}^{\infty} k[exp(-f_r/f)]^k + 2R^2\sum_{k=m}^{\infty}[exp(-f_r/f)]^k\right\}$$

$$= exp[-(m-1)f_r/f]\left\{\frac{m-(m-1)exp(-f_r/f)}{f[1-exp(-f_r/f)]}(\mu I + D) + 2R^2\right\} .$$ (10)

The average time per one block of subsequent successful frames and subsequent non-successful frames is given by equation

$$T_{ave1} = \sum_{k=1}^{\infty}\sum_{n=1}^{\infty} P_1(k,n)\frac{k+n}{f}$$

$$= \frac{1}{f}\sum_{k=1}^{\infty}[exp(-f_r/f)]^k\{k\sum_{n=1}^{\infty}[1-exp(-f_r/f)]^n + \sum_{n=1}^{\infty} n[1-exp(-f_r/f)]^n\}$$

$$= \frac{1-exp(-f_r/f)}{f \cdot exp(-f_r/f)}\left\{\sum_{k=1}^{\infty} k[exp(-f_r/f)]^k + \frac{1}{exp(-f_r/f)}\sum_{k=1}^{\infty}[exp(-f_r/f)]^k\right\}$$

$$= \frac{1}{f[1-exp(-f_r/f)]exp(-f_r/f)}$$ (11)

The square of the noise generation rate can be obtained by dividing (10) by (11) which equals

$$\breve{N}_1(m) = \frac{\tilde{N}_2(m)}{T_{ave1}} = exp(-mf_r/f)\{[m-(m-1)exp(-f_r/f)](\mu I + D) + f[1-exp(-f_r/f)]2R^2\} .$$ (12)

The signal generation rate can be obtained in a similar manner and it equals

$$\breve{S}_1(m) = [exp(-f_r/f)]^m[m-(m-1)exp(-f_r/f)]\mu I .$$ (13)

Thus the equation for SNR can be given in the following manner

$$SNR_{5NDCDS}(m) = \frac{\breve{S}_1 t}{\sqrt{\breve{N}_1 t}} = \frac{\mu I}{\sqrt{\mu I + D + R^2 f_{6eff}}}Q_5\sqrt{t} ,$$ (14)

wherein the effective read noise generation frequency is given by

$$f_{5eff}(m) = \frac{2f[1-exp(-f_r/f)]}{m-(m-1)exp(-f_r/f)} \, , \tag{15}$$

and the reduction factor of the SNR by

$$Q_5(m) = [exp(-f_r/2f)]^m \sqrt{m - (m-1)exp(-f_r/f)} \, . \tag{16}$$

In (15) and (16) the parameter m may have only positive integer values. The benefit in this procedure is that the higher the frame rate the higher the SNR. In addition the parameter m can be afterwards separately optimized for each pixel so that the SNR of each pixel is maximized.

HAND HELD CAMERA, MOTIONLESS SCENERY, LOSSLESS ROLL CORRECTION, & DCDS READOUT ONLY WHEN NECESSARY

When the scenery is motionless the SNR can be maximized in the lossless roll correction (accurate three axial angular velocity sensor & fast enough image capture) by performing readouts only when necessary meaning that the integration time of a frame is random. The SNR can be further maximized on pixel level according to the pixel specific signal generation rate by throwing away the information of frames that are shorter than a certain threshold value T_0. It should be also noted, that in lossless roll correction when the scenery is motionless it is preferable to utilize DCDS mode in a MIG sensor. Thus the same equations apply for both traditional and MIG image sensors.

The square of the noise according to the probability that the frame is at least T_0 long is

$$\widetilde{N}_3(T_0) = \int_{T_0}^{\infty} [(\mu I + D)t + R^2] f_r exp(-f_r t) dt$$

$$= exp(-f_r T_0) \left[\left(\frac{1}{f_r} + T_0 \right) (\mu I + D) + R^2 \right] . \tag{17}$$

With the help of (A1) the average time per one frame is

$$T_{ave2} = \int_0^{\infty} t f_r exp(-f_r t) dt = \frac{1}{f_r} \tag{18}$$

and thus the square of the noise generation rate can be expressed as

$$\widetilde{N}_2(T_0) = \frac{\widetilde{N}_3(T_0)}{T_{ave2}} = exp(-f_r T_0)[(1 + f_r T_0)(\mu I + D) + f_r R^2] \, . \tag{19}$$

The signal generation rate is, on the other hand,

$$\check{S}_2(T_0) = exp(-f_r T_0)(1 + f_r T_0)\mu I \, . \tag{20}$$

With the help of (19) and (20) the equation for SNR can be written followingly

$$SNR_{6DCDS}(T_0) = \frac{\check{S}_2 t}{\sqrt{\widetilde{N}_2 t}} = \frac{\mu I}{\sqrt{\mu I + D + R^2 f_{6eff}}} Q_6 \sqrt{t} \, , \tag{21}$$

wherein

$$f_{6eff}(T_0) = \frac{f_r}{1+f_r T_0},$$ (22)

$$Q_6(T_0) = exp(-f_r T_0/2)\sqrt{1+f_r T_0}.$$ (23)

wherein T_0 is a pixel specific parameter that can be afterwards optimized to maximize the SNR of the pixel and which obeys the inequality $0 \leq T_0$. The parameter f_r corresponds in this case to average frame rate.

TRIPOD, SUBJECTS IN THE SCENERY, & MULTIPLE DCDS READOUT

In this case it is assumed that the camera is attached to a tripod and that there are subjects in low light scenery. It is further assumed that flash is either not used or that at least some of the subjects stand out of flash's reach and thus a long exposure time is mandatory. The subjects are informed to stay as still as possible. In order to avoid image blur due to subject movements multiple frame method is used. Nevertheless some of the multiple frames would still be spoiled by small unintentional subject movements. It is further assumed that the images of subjects and beneficially of their individual body parts are formed in the final image by merging together areas from multiple frames by performing suitable rotations and translations.

In case a subject changes significantly its position or facial expression during the long exposure time image one would have more than one alternative for the specific position and/or facial expression to be selected into the final image. As a matter of fact it would actually be possible to combine a position and a facial expression from another position. The downside of the multiple positions is naturally that the more frequently a substantial change in the position or facial expression appears the shorter the exposure time and thus the lower the image quality.

In addition, in case of DCDS readout the image quality of a subject is given by the equation

$$SNR_{7DCDS,Subject} = \frac{\mu I_s t_s e^{-f_s/f}}{\sqrt{(\mu I_s + D + R^2 f)t_s e^{-f_s/f}}} = \frac{\mu I_s}{\sqrt{\mu I_s + D + R^2 f}} e^{-f_s/2f}\sqrt{t_s},$$ (24)

wherein f_s corresponds to the average frequency at which a frame is spoiled by subject's subtle movements, t_s corresponds to the time that the subject holds a certain position and/or a certain facial expression, and I_s corresponds to photon flux per pixel which originates from the subject, or beneficially from the face or from a certain body part of the subject. The optimal frame rate in equation (24) maximizes the SNR and corresponds to the zero value of the derivative of the equation (24) which is given by

$$f_{7optimum,Subject} = \frac{f_s}{2}\left(1 + \sqrt{1 + 4\frac{\mu I_s + D}{R^2 f_s}}\right)$$ (25)

The frame rate should preset according to f_s and μI_s. The former may vary a lot between different people. The latter may naturally vary a lot between different people and between green, red, blue, and possibly white pixels (people may be lit differently and the colors of their cloths may be different). Thus the task of finding an optimal frame rate is practically impossible.

The image quality of the background is given by the equation

$$SNR_{7DCDS,Background} = \frac{\mu I_b t}{\sqrt{(\mu I_b + D + R^2 f)t}} = \frac{\mu I_b}{\sqrt{\mu I_b + D + R^2 f}}\sqrt{t},$$ (26)

wherein t is the total exposure time and I_b is the photon flux per pixel from the background. One should note that the frame rate in (26) is optimized for subjects and not for background.

TRIPOD, SUBJECTS IN THE SCENERY, & MULTIPLE NDCDS READOUT

In this case the equation for SNR can be given in the following manner

$$SNR_{8NDCDS,Subject}(m_s) = \frac{\mu I_s}{\sqrt{\mu I_s + D + R^2 f_{8eff}}} Q_8 \sqrt{t_s} \, , \tag{27}$$

wherein the effective read noise generation frequency is given by

$$f_{8eff}(m_s) = \frac{2f[1 - exp(-f_s/f)]}{m_s - (m_s - 1)exp(-f_s/f)} \, , \tag{28}$$

and the SNR reduction factor by

$$Q_8(m_s) = [exp(-f_s/2f)]^{m_s} \sqrt{m_s - (m_s - 1)exp(-f_s/f)} \, . \tag{29}$$

In (28) and (29) the parameter m_s may have only positive integer values. The benefit in this procedure is that the higher the frame rate the higher the SNR. In addition the parameter m_s can be afterwards separately optimized for each pixel corresponding to subject area so that the SNR of each pixel is maximized.

The image quality for the background is given by the following equation

$$SNR_{8NDCDS,Background} = \frac{\mu I_b t}{\sqrt{(\mu I_b + D)t + R^2}} \approx \frac{\mu I_b}{\sqrt{\mu I_b + D}} \sqrt{t} \, . \tag{30}$$

HAND HELD CAMERA, SUBJECTS IN THE SCENERY, LOSSY ROLL CORRECTION, & MULTIPLE DCDS READOUT

In this case the image quality of a subject is given by the equation

$$SNR_{9DCDS,Subject} = \frac{\mu I_s t_s e^{-(f_r + f_s)/f}}{\sqrt{(\mu I_s + D + R^2 f)t_s e^{-(f_r + f_s)/f}}} = \frac{\mu I_s}{\sqrt{\mu I_s + D + R^2 f}} e^{-(f_r + f_s)/2f} \sqrt{t_s} \, , \tag{31}$$

The optimal frame rate in equation (31) maximizes the SNR and corresponds to the zero value of the derivative of the equation (31) which is given by

$$f_{9optimum,Subject} = \frac{f_r + f_s}{2}\left(1 + \sqrt{1 + 4\frac{\mu I_s + D}{R^2(f_r + f_s)}}\right) . \tag{32}$$

The frame rate should preset according to $f_r + f_s$ and μI_s. The former may vary a lot between different people. The latter may naturally vary a lot between different people and between green, red, blue, and possibly white pixels (people may be lit differently and the colors of their cloths may be different). Thus the task of finding an optimal frame rate is practically impossible.

The image quality of the background is given by the equation

$$SNR_{9DCDS,Background} = \frac{\mu I_b t e^{-f_r/f}}{\sqrt{(\mu I_b + D + R^2 f) t e^{-f_r/f}}} = \frac{\mu I_b}{\sqrt{\mu I_b + D + R^2 f}} e^{-f_r/2f} \sqrt{t} \,. \tag{33}$$

One should note that the frame rate in (33) is optimized for subjects and not for background.

HAND HELD CAMERA, SUBJECTS IN THE SCENERY, LOSSY ROLL CORRECTION, & MULTIPLE NDCDS READOUT

In this case the equation for SNR of subjects can be given in the following manner

$$SNR_{10NDCDS,Subject}(m_s) = \frac{\mu I_s}{\sqrt{\mu I_s + D + R^2 f_{10eff,Subject}}} Q_{10,Subject} \sqrt{t_s} \,, \tag{34}$$

wherein the effective read noise generation frequency is given by

$$f_{10eff,Subject}(m_s) = \frac{2f\{1 - exp[-(f_r + f_s)/f]\}}{m_s - (m_s - 1) exp[-(f_r + f_s)/f]} \,, \tag{35}$$

and the SNR reduction factor by

$$Q_{10,Subject}(m_s) = \{exp[-(f_r + f_s)/2f]\}^{m_s} \sqrt{m_s - (m_s - 1) exp[-(f_r + f_s)/f]} \,. \tag{36}$$

In (35) and (36) the parameter m_s may have only positive integer values. The benefit in this procedure is that the higher the frame rate the higher the SNR. In addition the parameter m_s can be afterwards separately optimized for each pixel corresponding to subject area so that the SNR of each pixel is maximized.

The equation for SNR of the background can be given in the following manner

$$SNR_{10NDCDS,Background}(m_b) = \frac{\mu I_b}{\sqrt{\mu I_b + D + R^2 f_{10eff,Background}}} Q_{10,Background} \sqrt{t} \,, \tag{37}$$

wherein the effective read noise generation frequency is given by

$$f_{10eff,Background}(m_b) = \frac{2f[1 - exp(-f_r/f)]}{m_b - (m_b - 1) exp(-f_r/f)} \,, \tag{38}$$

and the SNR reduction factor by

$$Q_{10,Background}(m_b) = [exp(-f_r/2f)]^{m_b} \sqrt{m_b - (m_b - 1) exp(-f_r/f)} \,. \tag{39}$$

In (38) and (39) the parameter m_b may have only positive integer values. The benefit in this procedure is that the higher the frame rate the higher the SNR. In addition the parameter m_b can be afterwards separately optimized for each pixel corresponding to background area so that the SNR of each pixel is maximized.

HAND HELD CAMERA, SUBJECTS IN THE SCENERY, LOSSLESS ROLL CORRECTION, & MULTIPLE DCDS READOUT

In this case it is assumed that the interval between frames is $1/f$ when no roll correction is required and that the interval shorter if a lossless roll correction is required before $1/f$. It is also assumed that if the length of the frame corresponding to lossless correction is below a threshold T_{00} it will be thrown away.

The average value of the square of the noise according to the probability that lossless roll correction happens during the time period $T_{s00} \leq t \leq 1/f$ before the frame is spoiled by subject movement can be given by the following equation

$$\widetilde{N}_4(T_{s00}) = \int_{T_{s00}}^{1/f}[(\mu I_s + D)t + R^2]f_r exp(-f_r t)\int_t^\infty f_s exp(-f_s t')dt' \, dt$$

$$= f_r\left\{(\mu I_s + D)\int_{T_{s00}}^{1/f} t \cdot exp[-(f_r + f_s)t]dt + R^2 \int_{T_{s00}}^{1/f} exp[-(f_r + f_s)t]dt\right\}$$

$$= \frac{f_r}{f_r+f_s}\left\{\begin{array}{c}\left\{\left(\frac{1}{f_r+f_s} + T_{s00}\right)exp[-(f_r + f_s)T_{s00}] - \left(\frac{1}{f_r+f_s} + \frac{1}{f}\right)exp[-(f_r + f_s)/f]\right\}(\mu I_s + D) \\ +\{exp[-(f_r + f_s)T_{s00}] - exp[-(f_r + f_s)/f]\}R^2\end{array}\right\} . \quad (40)$$

The square of the noise according to the probability that the next frame is reached before lossless roll correction takes place and before the frame is spoiled by subject movement is given by the following equation

$$\widetilde{N}_5 = \left(\frac{\mu I_s + D}{f} + R^2\right)\int_{1/f}^\infty f_r exp(-f_r t)dt \int_{1/f}^\infty f_s exp(-f_s t)dt$$

$$= \left(\frac{\mu I_s + D}{f} + R^2\right)exp[-(f_r + f_s)/f] . \quad (41)$$

The average time of one frame is given by equation

$$T_{ave3} = \int_0^{1/f} tf_r exp(-f_r t)dt + \frac{1}{f}\int_{1/f}^\infty f_r exp(-f_r t)dt$$

$$= \frac{1}{f_r}[1 - exp(-f_r/f)] . \quad (42)$$

Thus the square of the noise generation rate can be expressed in the following manner

$$\widetilde{N}_3(T_{s00}) = \frac{\widetilde{N}_4 + \widetilde{N}_5}{T_{ave2}}$$

$$= \frac{f_r}{1 - exp(-f_r/f)}$$
$$\times\left\{\begin{array}{c}\left\{\frac{f_r}{(f_r+f_s)^2}[1 + (f_r + f_s)T_{s00}]exp[-(f_r + f_s)T_{s00}] + \left[\frac{f_s}{f(f_r+f_s)} - \frac{f_r}{(f_r+f_s)^2}\right]exp[-(f_r + f_s)/f]\right\}(\mu I_s + D) \\ +\left\{\frac{f_r}{f_r+f_s}exp[-(f_r + f_s)T_{s00}] + \frac{f_s}{f_r+f_s}exp[-(f_r + f_s)/f]\right\}R^2\end{array}\right\} . \quad (43)$$

The equation for the signal generation rate can be obtained in a similar manner and equals

$$\widecheck{S}_3(T_{s00}) = \frac{f_r}{1 - exp(-f_r/f)}\left\{\frac{f_r}{(f_r+f_s)^2}[1 + (f_r + f_s)T_{s00}]exp[-(f_r + f_s)T_{s00}] + \left[\frac{f_s}{f(f_r+f_s)} - \frac{f_r}{(f_r+f_s)^2}\right]exp[-(f_r + f_s)/f]\right\}\mu I_s . \quad (44)$$

Consequently the equation for the SNR can be expressed as

$$SNR_{11DCDS,Subject}(T_{s00}) = \frac{\check{S}_2 t_s}{\sqrt{\tilde{N}_2 t_s}} = \frac{\mu I_s}{\sqrt{\mu I_s + D + R^2 f_{11eff,Subject}}} Q_{11Subject} \sqrt{t_s} \, , \tag{45}$$

wherein the effective read noise generation frequency is given by

$$f_{11eff,Subject}(T_{s00}) = \frac{\frac{f_r}{f_r+f_s}exp[-(f_r+f_s)T_{s00}]+\frac{f_s}{f_r+f_s}exp[-(f_r+f_s)/f]}{\frac{f_r}{(f_r+f_s)^2}[1+(f_r+f_s)T_{s00}]exp[-(f_r+f_s)T_{s00}]+\left[\frac{f_s}{f(f_r+f_s)}-\frac{f_r}{(f_r+f_s)^2}\right]exp[-(f_r+f_s)/f]} \, , \tag{46}$$

and the reduction factor of the SNR by

$$Q_{11Subject}(T_{s00}) = \sqrt{\frac{f_r}{1-exp(-f_r/f)}\left\{\frac{f_r}{(f_r+f_s)^2}[1+(f_r+f_s)T_{s00}]exp[-(f_r+f_s)T_{s00}]+\left[\frac{f_s}{f(f_r+f_s)}-\frac{f_r}{(f_r+f_s)^2}\right]exp[-(f_r+f_s)/f]\right\}} , \tag{47}$$

wherein T_{s00} is a pixel specific parameter that can be afterwards optimized to maximize the SNR of the pixel and which obeys the inequality $0 \leq T_{s00} \leq 1/f$.

As already stated previously the frame rate should preset according to $f_r + f_s$ and μI_s. The former term may vary a lot between different people. The latter term may naturally vary a lot between different people and between green, red, blue, and possibly white pixels (people may be lit differently and the colors of their cloths may be different). Thus the task of finding an optimal frame rate for (45), (46), and (47) is practically an impossible task.

The SNR of the background can be obtained by setting f_s to zero in (45), (46), and (47) which yields

$$SNR_{11DCDS,Background}(T_{b00}) = \frac{\mu I_b}{\sqrt{\mu I_b + D + R^2 f_{11eff,Background}}} Q_{11Background} \sqrt{t} \, , \tag{48}$$

wherein the effective read noise generation frequency is given by

$$f_{11eff,Background}(T_{b00}) = \frac{f_r exp(-f_r T_{b00})}{(1+f_r T_{b00})exp(-f_r T_{b00})-exp(-f_r/f)} \, , \tag{49}$$

and the reduction factor of the SNR by

$$Q_{11Background}(T_{b00}) = \sqrt{\frac{(1+f_r T_{b00})exp(-f_r T_{b00})-exp(-f_r/f)}{1-exp(-f_r/f)}} \, . \tag{50}$$

wherein T_{b00} is a pixel specific parameter that can be afterwards optimized to maximize the SNR of the pixel and which obeys the inequality $0 \leq T_{b00} \leq 1/f$.

HAND HELD CAMERA, SUBJECTS IN THE SCENERY, LOSSLESS ROLL CORRECTION, & MULTIPLE NDCDS READOUT

In this case the frame rate is always synchronized to the previous lossless roll correction, i.e., the readout corresponding to the lossless roll correction is followed by frames that are placed at an interval of $1/f$ until another readout corresponding to roll correction is required. In other words, the exposure time of the frame corresponding to the roll correction is shorter than $1/f$ but the exposure time of other frames is $1/f$.

One should also note that in between two roll corrections there may be multiple subsequent time periods during which the movement of a subject does not cause image blur at the area of the image wherein the subject is placed. Such time periods are referred to as local blur free periods. As already stated before in between two local blur free periods there is always one local frame which information is thrown away in order to remove correlation between subsequent local blur free periods. In addition the first local frame of the local blur free period is subtracted from the last local frame in order to simplify the equation and the information corresponding to intermediate local frames is thrown away. The downside of afore described procedure is, however, that the SNR is somewhat underestimated.

The average value of the square of the noise according to the probability that
-during the time period between two roll corrections a subject does not cause image blur, and that
-the time period between two roll corrections is at least as long as T_{s1}
is given by the equation

$$\tilde{N}_6(T_{s1}) = \int_{T_{s1}}^{\infty} [(\mu I_s + D)t + R^2] f_r exp(-f_r t) \int_t^{\infty} f_s exp(-f_s t') dt' \, dt$$

$$= (\mu I_s + D) f_r \int_{T_{s1}}^{\infty} t \cdot exp[-(f_r + f_s)t] dt + R^2 f_r \int_{T_{s1}}^{\infty} exp[-(f_r + f_s)t] dt$$

$$= \frac{f_r}{f_r + f_s} exp[-(f_r + f_s)T_{s1}] \left[\left(\frac{1}{f_r + f_s} + T_{s1} \right) (\mu I_s + D) + R^2 \right], \qquad (51)$$

wherein the pixel specific optimization parameter $T_{s1} \geq 0$.

The average value of the square of the noise according to the probability that
-the local blur free period starts at a time point n/f after the lossless roll correction (i.e., information of the previous frame is thrown away), that
-the local blur free period ends at roll correction (i.e., there is no subject induced image blur in between time point n/f and a lossless roll correction), and that
-the local blur free period between n/f and the roll correction is at least as long as T_{s2}
is given by the equation

$$\tilde{N}_7(n, T_{s2}) = \int_{(n-1)/f}^{n/f} f_s exp\{-f_s[t - (n-1)/f]\} dt$$
$$\times \int_{n/f+T_{s2}}^{\infty} [(\mu I_s + D)(t - n/f) + 2R^2] f_r exp(-f_r t) \int_t^{\infty} f_s exp[-f_s(t' - n/f)] dt' \, dt$$

$$= f_r[1 - exp(-f_s/f)] exp(n f_s/f)$$
$$\times \left\{ 2R^2 \int_{n/f+T_{s2}}^{\infty} exp[-(f_r + f_s)t] dt + (\mu I_s + D) \int_{n/f+T_{s2}}^{\infty} (t - n/f) exp[-(f_r + f_s)t] dt \right\}$$

$$= \frac{f_r}{f_r+f_s}[1 - exp(-f_s/f)]exp(-nf_r/f)exp[-(f_r+f_s)T_{s2}]$$
$$\times \left[\left(\frac{1}{f_r+f_s} + T_{s2}\right)(\mu I_s + D) + 2R^2\right]. \tag{52}$$

Thus the average value of the square of the noise according to the probability that
-the local blur free period starts at a time point n/f after the lossless roll correction (i.e., information of the previous frame is thrown away), that
-the local blur free period ends at roll correction before the subtle movements of the subject introduce image blur, and that
-the local blur free period is at least as long as T_{s2}
is given by the equation

$$\tilde{N}_8(T_{s2}) = \sum_{n=1}^{\infty} \tilde{N}_7(n, T_{s2})$$

$$= \frac{f_r}{f_r+f_s}[1 - exp(-f_s/f)]exp[-(f_r+f_s)T_{s2}]$$
$$\times \left[\left(\frac{1}{f_r+f_s} + T_{s2}\right)(\mu I_s + D) + 2R^2\right]\sum_{n=1}^{\infty}[exp(-f_r/f)]^n$$

$$= \frac{f_r}{f_r+f_s}\frac{exp(-f_r/f)}{1-exp(-f_r/f)}[1 - exp(-f_s/f)]exp[-(f_r+f_s)T_{s2}]\left[\left(\frac{1}{f_r+f_s} + T_{s2}\right)(\mu I_s + D) + 2R^2\right], \tag{53}$$

wherein the pixel specific optimization parameter $T_{s2} \geq 0$.

The probability according to that
-the local blur free period starts from lossless roll correction, and that
-the local blur free period ends at a time point of k/f before another lossless roll correction takes place
is given by the equation

$$P_3(k) = \int_{k/f}^{(k+1)/f} f_s exp(-f_s t) \int_t^{\infty} f_r exp(-f_r t')dt' \, dt$$

$$= \frac{f_s}{f_r+f_s}\{1 - exp[(f_r+f_s)/f]\}\{exp[(f_r+f_s)/f]\}^k \tag{54}$$

The average value of the square of the noise according to probability (54) is

$$\tilde{N}_9(k) = P_3(k)\left[(\mu I_s + D)\frac{k}{f} + R^2\right]$$

$$= \frac{f_s}{f_r+f_s}\{1 - exp[(f_r+f_s)/f]\}\{exp[(f_r+f_s)/f]\}^k\left[(\mu I_s + D)\frac{k}{f} + R^2\right]. \tag{55}$$

The average value of the square of the noise according to the probability that
-the local blur free period starts from lossless roll correction, that
-the local blur free period ends before another lossless roll correction takes place, and that
-the local blur free period is at least as long as m_{s1}/f , i.e. $m_{s1} \leq k$
is given by the equation

$$\tilde{N}_{10}(m_{s1}) = \sum_{k=m_{s1}}^{\infty} \tilde{N}_9(k)$$

$$= \frac{f_s}{f_r+f_s}\{1 - exp[-(f_r+f_s)/f]\}$$
$$\times \left\{\frac{\mu I_s + D}{f}\sum_{k=m_{s1}}^{\infty} k\{exp[-(f_r+f_s)/f]\}^k + R^2\sum_{k=m_{s1}}^{\infty}\{exp[-(f_r+f_s)/f]\}^k\right\}$$

$$= \frac{f_s}{f_r+f_s} \{exp[-(f_r+f_s)/f]\}^{m_{s1}} \left\{ \frac{m_{s1}-(m_{s1}-1)exp[-(f_r+f_s)/f]}{f\{1-exp[-(f_r+f_s)/f]\}} (\mu I_s + D) + R^2 \right\}, \tag{56}$$

wherein the pixel specific optimization parameter m_{s1} is a positive integer.

The probability according to that
-the local blur free period starts at a time point n/f after the lossless roll correction (i.e., information of the previous frame is thrown away), and that
-the local blur free period ends at a time point k/f before another lossless roll correction takes place
is given by the equation

$$P_3(n,k) = \int_{(n-1)/f}^{n/f} f_s exp\{-f_s[t-(n-1)/f]\}dt$$
$$\times \int_{(n+k)/f}^{(n+k+1)/f} f_s exp[-f_s(t-n/f)] \int_t^\infty f_r exp(-f_r t')dt'\, dt$$

$$= \frac{f_s}{f_r+f_s}[1 - exp(-f_s/f)]\{1-exp[-(f_r+f_s)/f]\}[exp(-f_r/f)]^n\{exp[-(f_r+f_s)/f]\}^k . \tag{57}$$

The average value of the square of the noise according to the probability (57) is

$$\widetilde{N}_{11}(n,k) = P_3(n,k)\left[(\mu I_s + D)\frac{k}{f} + 2R^2\right]$$

$$= \frac{f_s}{f_r+f_s}[1 - exp(-f_s/f)]\{1-exp[-(f_r+f_s)/f]\}$$
$$\times [exp(-f_r/f)]^n\{exp[-(f_r+f_s)/f]\}^k \left[(\mu I_s + D)\frac{k}{f} + 2R^2\right]. \tag{58}$$

The average value of the square of the noise according to the probability that
-the local blur free period does not start from lossless roll correction, that
-the local blur free period does not end at lossless roll correction, and that
-the local blur free period is at least as long as m_{s2}/f , i.e. $m_{s2} \le k$
is given by the equation

$$\widetilde{N}_{12}(m_{s2}) = \sum_{n=1}^\infty \sum_{k=m_{s2}}^\infty \widetilde{N}_{11}(n,k)$$

$$= \frac{f_s}{f_r+f_s}[1 - exp(-f_s/f)]\{1-exp[-(f_r+f_s)/f]\}\sum_{n=1}^\infty [exp(-f_r/f)]^n$$
$$\times \left\{ \frac{\mu I_s+D}{f}\sum_{k=m_{s2}}^\infty k\{exp[-(f_r+f_s)/f]\}^k + 2R^2\sum_{k=m_{s2}}^\infty k\{exp[-(f_r+f_s)/f]\}^k \right\}$$

$$= \frac{f_s}{f_r+f_s}\frac{exp(-f_r/f)}{1-exp(-f_r/f)}[1 - exp(-f_s/f)]$$
$$\times \{exp[-(f_r+f_s)/f]\}^{m_{s2}} \left\{ \frac{m_{s2}-(m_{s2}-1)exp[-(f_r+f_s)/f]}{f\{1-exp[-(f_r+f_s)/f]\}} (\mu I_s + D) + 2R^2 \right\}, \tag{59}$$

wherein the pixel specific optimization parameter m_{s2} is a positive integer.

The average time between two lossless roll corrections is

$$T_{ave4} = \int_0^\infty t f_r exp(-f_r t)dt = \frac{1}{f_r}, \tag{60}$$

and thus the square of the noise generation rate and the signal generation rate corresponding to (51) are given by

$$\breve{N}_4(T_{s1}) = \frac{\tilde{N}_6(T_{s1})}{T_{ave3}} = W_1(T_{s1})(\mu I_s + D) + V_1(T_{s1})R^2 \,, \tag{61}$$

$$\breve{S}_4(T_{s1}) = W_1(T_{s1})\mu I_s \,, \tag{62}$$

wherein

$$V_1(T_{s1}) = \frac{f_r^2}{f_r+f_s} exp[-(f_r + f_s)T_{s1}] \tag{63}$$

$$W_1(T_{s1}) = V_1(T_{s1})\left(\frac{1}{f_r+f_s} + T_{s1}\right), \tag{64}$$

The square of the noise generation rate and the signal generation rate corresponding to (53) are given by

$$\breve{N}_5(T_{s2}) = \frac{\tilde{N}_8(T_{s2})}{T_{ave3}} = W_2(T_{s2})(\mu I_s + D) + V_2(T_{s2})2R^2 \,, \tag{65}$$

$$\breve{S}_5(T_{s2}) = W_2(T_{s2})\mu I_s \,, \tag{66}$$

wherein

$$V_2(T_{s2}) = \frac{f_r^2}{f_r+f_s}\frac{exp(-f_r/f)}{1-exp(-f_r/f)}[1 - exp(-f_s/f)]exp[-(f_r + f_s)T_{s2}] \,. \tag{67}$$

$$W_2(T_{s2}) = V_2(T_{s2})\left(\frac{1}{f_r+f_s} + T_{s2}\right), \tag{68}$$

The square of the noise generation rate and the signal generation rate corresponding to (56) are given by

$$\breve{N}_6(m_{s1}) = \frac{\tilde{N}_{10}(m_{s1})}{T_{ave3}} = W_3(m_{s1})(\mu I_s + D) + V_3(m_{s1})R^2 \,, \tag{69}$$

$$\breve{S}_6(m_{s1}) = W_3(m_{s1})\mu I_s \,. \tag{70}$$

wherein

$$V_3(m_{s1}) = \frac{f_r f_s}{f_r+f_s}\{exp[-(f_r + f_s)/f]\}^{m_{s1}} \,. \tag{71}$$

$$W_3(m_{s1}) = V_3(m_{s1})\frac{m_{s1}-(m_{s1}-1)exp[-(f_r+f_s)/f]}{f\{1-exp[-(f_r+f_s)/f]\}} \,, \tag{72}$$

The square of the noise generation rate and the signal generation rate corresponding to (59) are given by

$$\breve{N}_7(m_{s2}) = \frac{\tilde{N}_{12}(m_{s2})}{T_{ave3}} = W_4(m_{s2})(\mu I_s + D) + V_4(m_{s2})2R^2 \,, \tag{73}$$

$$\breve{S}_7(m_{s2}) = W_4(m_{s2})\mu I_s \,, \tag{74}$$

wherein

$$V_4(m_{s2}) = \frac{f_r f_s}{f_r+f_s}\frac{exp(-f_r/f)}{1-exp(-f_r/f)}[1 - exp(-f_s/f)]\{exp[-(f_r + f_s)/f]\}^{m_{s2}} \,. \tag{75}$$

$$W_4(m_{s2}) = V_4(m_{s2})\frac{m_{s2}-(m_{s2}-1)exp[-(f_r+f_s)/f]}{f\{1-exp[-(f_r+f_s)/f]\}} \,, \tag{76}$$

The SNR corresponding to (61) – (76) is given by equation

$$SNR_{12NDCDS,Subject}(T_{s1}, T_{s2}, m_{s1}, m_{s2}) = \frac{[\check{S}_4(T_{s1}) + \check{S}_5(T_{s2}) + \check{S}_6(m_{s1}) + \check{S}_7(m_{s2})]t_s}{\sqrt{[\check{N}_4(T_{s1}) + \check{N}_5(T_{s2}) + \check{N}_6(m_{s1}) + \check{N}_7(m_{s2})]t_s}}$$

$$= \frac{\mu I_s}{\sqrt{\mu I_s + D + R^2 f_{12eff,Subject}}} Q_{12Subject}\sqrt{t_s}, \tag{77}$$

wherein the effective read noise generation frequency is given by

$$f_{12eff,Subject}(T_{s1}, T_{s2}, m_{s1}, m_{s2}) = \frac{V_1(T_{s1}) + 2V_2(T_{s2}) + V_3(m_{s1}) + 2V_4(m_{s2})}{W_1(T_{s1}) + W_2(T_{s2}) + W_3(m_{s1}) + W_4(m_{s2})}, \tag{78}$$

and the reduction factor of the SNR by

$$Q_{12Subject}(T_{s1}, T_{s2}, m_{s1}, m_{s2}) = \sqrt{W_1(T_{s1}) + W_2(T_{s2}) + W_3(m_{s1}) + W_4(m_{s2})}. \tag{79}$$

The benefit of (77) is that the SNR is the higher the bigger the frame rate and that the parameters T_{s1}, T_{s2}, m_{s1}, and m_{s2} can be separately optimized for each pixel.

The image quality in the background can be obtained by setting f_s to zero in (61) – (76) which results in the following equation

$$SNR_{12NDCDS,Background}(T_{b1}) = \frac{\mu I_b}{\sqrt{\mu I_s + D + R^2 f_{12eff,Background}}} Q_{12Background}\sqrt{t} \tag{80}$$

wherein

$$f_{12eff,Background}(T_{b1}) = \frac{f_r}{1 + f_r T_{b1}}, \tag{81}$$

$$Q_{12Background}(T_{b1}) = exp(-f_r T_{b1}/2)\sqrt{1 + f_r T_{b1}}, \tag{82}$$

wherein T_{b1} corresponds to T_{s1}. The equations (81) and (82) are exactly the same as the equations (23) and (24) just as they should be.

APPENDIX

$$\frac{d}{dx}F(x) = f(x), \quad \frac{d}{dx}G(x) = g(x): \quad \int f(x)G(x)dx = F(x)G(x) - \int F(x)g(x)dx \tag{A1}$$

$$|q| < 1: \quad \sum_{k=m}^{\infty} kq^k = q\sum_{k=m}^{\infty} kq^{k-1} = q\frac{\partial}{\partial q}\sum_{k=m}^{\infty} q^k = q\frac{\partial}{\partial q}\frac{q^m}{1-q} = q\left(\frac{mq^{m-1}}{1-q} + \frac{q^m}{(1-q)^2}\right) = \frac{q^m}{(1-q)^2}[m - (m-1)q] \tag{A2}$$

$$\lim_{f_r \to 0} \frac{f_r}{1-exp(-f_r/f)} = f\lim_{f_r \to 0} \frac{f_r}{f}\frac{exp(f_r/f)}{exp(f_r/f)-1} = f\lim_{y \to 0} y\frac{exp(y)}{exp(y)-1} = f\lim_{y \to 0}\frac{y}{exp(y)-1} = f\lim_{y \to 0}\frac{y}{1+y+\Delta(y^2)-1}$$

$$= f\lim_{y \to 0}\frac{1}{1+\Delta(y)} = f \tag{A3}$$

$$\lim_{f \to 0}\frac{exp(-f_r/f)}{f} = \frac{1}{f_r}\lim_{f \to 0}\frac{f_r/f}{exp(f_r/f)} = \frac{1}{f_r}\lim_{y \to \infty}\frac{y}{exp(y)} = 0 \tag{A4}$$

$$\lim_{f \to \infty}\frac{1}{f\{1-exp[-(f_r+f_s)/f]\}} = \frac{1}{f_r+f_s}\lim_{f \to \infty}\frac{f_r+f_s}{f}\frac{exp[(f_r+f_s)/f]}{exp[(f_r+f_s)/f]-1} = \frac{1}{f_r+f_s}\lim_{y \to 0} y\frac{exp(y)}{exp(y)-1}$$

$$= \frac{1}{f_r+f_s}\lim_{y \to 0}\frac{y}{exp(y)-1} = \frac{1}{f_r+f_s}\lim_{y \to 0}\frac{y}{1+y+\Delta(y^2)-1} = \frac{1}{f_r+f_s}\lim_{y \to 0}\frac{1}{1+\Delta(y)} = \frac{1}{f_r+f_s} \tag{A5}$$